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A paradox for traffic dynamics in complex networks with ATIS

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Abstract

In this work, we study the statistical properties of traffic (e.g., vehicles) dynamics in complex networks, by introducing advanced transportation information systems (ATIS). The ATIS can provide the information of traffic flow pattern throughout the network and have an obvious effect on path routing strategy for such vehicles equipped with ATIS. The ATIS can be described by the understanding of link cost functions. Different indices such as efficiency and system total cost are discussed in depth. It is found that, for random networks (scale-free networks), the efficiency is effectively improved (decreased) if ATIS is properly equipped; however the system total cost is largely increased (decreased). It indicates that there exists a paradox between the efficiency and system total cost in complex networks. Furthermore, we report the simulation results by considering different kinds of link cost functions, and the paradox is recovered. Finally, we extend our traffic model, and also find the existence of the paradox.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the last few years, complex networked systems have attracted much attention, including the protein–protein interaction networks [1, 2], food webs [3, 4], scientific collaboration networks [5–7], sexual relations [8], transportation networks [9, 10], the World Wide Web [11–13], etc. Furthermore, some typical review works on complex networks can be referred to [14–16]. It has been well proved that the topological features of underling interaction networks have great impacts on the final outcomes of the dynamics taking place on them. For instance, the scale-free topology of a network gives rise to a robust behavior against random failures and is fragile for aimed attack [17]; the highly clustered networks are found to more easily spread epidemics [18].

Among the dynamical processes taking place on complex networks [15, 16, 19], the packets transport in the Internet [20–24] or vehicles transport in the transportation networks [25–27], may be of practical importance, since it plays a more and more important role in our daily life. Researchers have proposed some models to mimic the traffic on complex networks by introducing the random generation and the routing of packets [28–32]. Recently, Zheng *et al* [26] and Zhao *et al* [27] introduced user equilibrium and system optimum [33] to determine the traffic flow pattern throughout the transportation network, and studied the traffic dynamics in various kinds of complex networks.

In many previous studies, each object (e.g., information, vehicles, etc) seems to have the same information of traffic flow distribution when determining the shortest path routing strategy. In this paper, we introduce advanced transportation information systems (ATIS) and investigate the effect of ATIS on traffic dynamics in complex networks. The ATIS can provide the information of traffic flow distribution and have an obvious effect on path routing strategy for such vehicles equipped with ATIS. The ATIS can be described by the understanding of link travel times, which are given by link cost functions. Then it gives rise to a question, i.e., in order to improve the performance (e.g., system total cost, efficiency, etc) of the network, how many vehicles should be equipped with ATIS. It is simply assumed that the ATIS is controlled by parameter $q (0 \leqslant q \leqslant 1)$[3]. As will be shown below, a paradox between the efficiency and system total cost in complex networks is found.

The rest of this paper is organized as follows. Section 2 provides a brief introduction about a traffic dynamics model. Simulations and results are given in section 3. Finally, some summaries and conclusions are shown in section 4.

2. Traffic dynamics model

Networks comprise nodes and links, which represent the individuals and their interactions, respectively. We consider the network with N nodes, and many vehicles simultaneously travel from different origins to different destinations (all nodes are both origins and destinations), in terms of minimum-cost routes. For simplicity, we assume that between each origin–destination pair r–s, the unity traffic volume travels from r to s. We introduce ATIS to influence the path routing strategy, which is controlled by parameter q. Then it is simply assumed that the traffic flow pattern is determined by the following rules:

- For q fraction of the traffic volume, vehicles travel along the minimum-cost routes when there is no flow. In this case, we assume that each link's cost is equal to 1. In other words, these vehicles determine their routes according to the network topology.
- Update the traffic flow distribution, and calculate the link travel times (i.e., link costs) in terms of link cost functions.
- For the rest $1 - q$ fraction of the traffic volume, vehicles travel from the same origins to destinations by choosing different routes in terms of updated link costs.

In our simulations, the shortest paths mentioned above are determined by the famous Floyd's algorithm. It is clear that, the unity traffic volume is divided into two different parts, i.e., not equipped with ATIS and equipped with ATIS. The vehicles equipped with AITS can get the information of the traffic flow distribution of the vehicles not equipped with ATIS, and may

[3] For $1 - q$ fraction of the traffic volume, vehicles are equipped with ATIS. It is simply assumed that these vehicles can get the information of the rest traffic volume.

have the advantage to choose better routes to avoid congestion. According to rule (i), such shortest path routing strategy is the most typically used in previous works, and the load on the node is equivalent to the betweenness centrality [34]. Congestion effects [33] described by rule (ii) play an important role in determining traffic flow pattern throughout the network.

For the link cost functions, the BPR (the Bureau of Public Roads) formula [35] is the most widely used, expressed as

$$w_{ij} = w_{ij}^0 \left(1 + a\left(\frac{x_{ij}}{C_{ij}}\right)^b\right), \tag{1}$$

where $a = 0.15$ and $b = 4$ are typically used. w_{ij}^0 denotes the travel time on the link (i, j) when there is no flow, which is also called as free-flow cost. x_{ij} is the traffic flow on the link (i, j), and C_{ij} is the 'practical capacity' of link (i, j).

3. Simulations and results

For complex networks, we focus on scale-free networks and random networks. In our simulations, the network size is $N = 1000 \sim 5000$, and the average degree is $\langle k\rangle = 6$. Random networks can be generated by the binomial model [14], where each pair of nodes is linked with probability $\frac{\langle k\rangle}{N}$. To generate scale-free networks, we use the standard Barábasi–Albert (BA) scale-free model [36]. Two different indices used in our simulations are system total cost STC [33] and efficiency E [37], defined as follows:

$$STC = \sum_{(i,j)} x_{ij} w_{ij} \tag{2}$$

$$E = \frac{2}{N(N-1)} \sum_{i,j} \frac{1}{\omega_{ij}}, \tag{3}$$

where ω_{ij} is the minimum cost for traveling from node i to node j.

ATIS can be described by link cost functions. In the following, we perform our simulations by considering different kinds of link cost functions, i.e., homogeneous link cost functions and heterogeneous link cost functions. All the resulting data are averaged over ten realizations.

3.1. With homogeneous link cost functions

In this section, $w_{ij}^0 = 1$ and $C_{ij} = 300$ for each link are typically used in the BPR functions.

In figure 1, we show the relationship between efficiency E and parameter q in complex networks. From figure 1, one can see that for scale-free networks, the efficiency is sharply decreased due to the effect of ATIS, while for random networks, the efficiency is largely improved if the ATIS is properly equipped. The optimal value of the parameter q for random networks is numerically estimated to be about $q_1 = 0.21$, and the efficiency is improved more than three times, as shown in the inset of figure 1. Similarly, we show the plot of system total cost STC against parameter q in figure 2. It is clear that for scale-free networks, it performs a 'U' form, while for random networks, it exhibits a 'bell' shape, and it approaches the peak when $q = q_2$. The critical value of parameter q, q_2 is numerically estimated to be about $q_2 = 0.5$, as shown in the inset of figure 2.

As mentioned above, it is obvious that the effect of network structure plays an important role in determining traffic dynamics in complex networks. In scale-free networks, there exist some 'hub' nodes, which are to handle more traffic flows. In our model, these 'hub' nodes

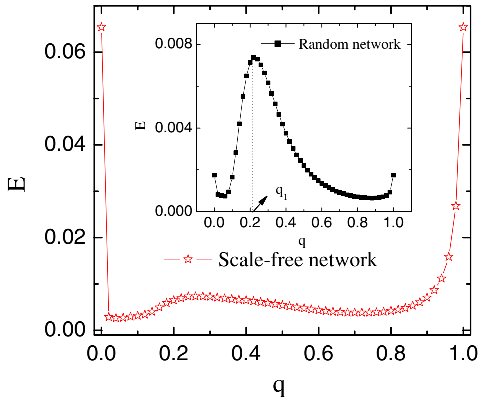

Figure 1. Efficiency E as a function of q in a scale-free network. In the inset we show the same plot for a random network. The network size is $N = 3000$.

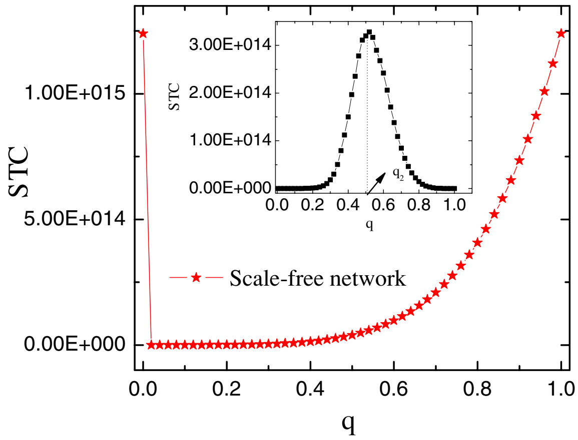

Figure 2. System total cost STC as a function of q in a scale-free network. In the inset, we show the same plot for a random network. The network size is $N = 3000$.

are heavily congested as compared with other nodes after rules (i) and (ii). Then the vehicles equipped with ATIS may make a detour to avoid these 'hub' nodes. As a result, the efficiency of the network is largely decreased. On the other hand, the burden of these 'hub' nodes is largely relieved. Then the distribution of the traffic flow becomes less heterogeneous, causing that the system total cost is also decreased. Contrarily, in random networks, due to the effect of ATIS, the efficiency of the network is effectively improved; however, the system total cost is increased at the same time. So, it indicates that there exists a paradox between the efficiency and system total cost in complex networks. The reason may be due to the conflicts between individual view and systemic view. Actually, vehicles determine the path routing in terms of the minimum cost. Then the efficiency is effectively improved in the individual view. Generally, the sum of individual optimum is not equivalent to system optimum. As a result, the system total cost is not decreased in systemic view, though the efficiency is improved.

In figure 3, we report the diagram of system total cost against efficiency, since there exists a paradox between them. Each point in the diagram is for the same value of parameter

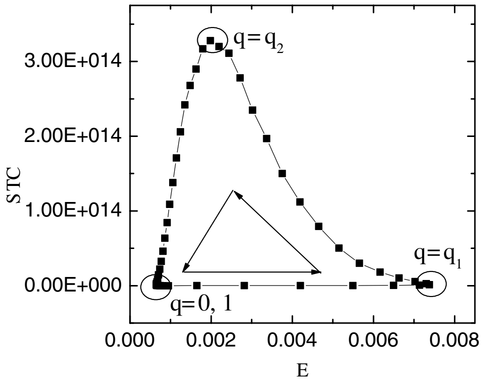

Figure 3. System total cost STC as a function of efficiency E in a random network with $N = 3000$.

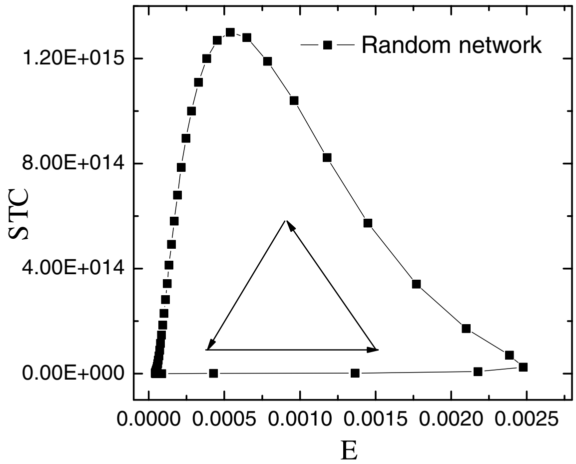

Figure 4. System total cost STC as a function of efficiency E in a random network with $N = 5000$.

q. And all the points are connected as the increase of parameter q. In this way a loop can be traced (arrows in figure 3). Moreover, there exist three critical points, corresponding to $q = 0, 1$, $q = q_1$ and $q = q_2$, respectively. It is clear that a paradox is found when $0 < q < q_1$ or $q_2 < q < 1$. In order to test the finite-size effect, we simulate a system with a bigger size. The simulation gives a similar result, as shown in figure 4. However, values of q_1 and q_2 are different from that for the case of smaller network size. As will be shown below, q_1 and q_2 depend on the parameters (e.g., link flows or 'practical capacity' of the links) in link cost functions, where the volume of link flow distribution is seriously influenced by the network size.

In reality, it is interesting to observe the effect of noise on the dynamical process. In [38], when noise is introduced into the nonlinear dynamical system, it has been shown that noise changes the singularity at a special time to a statistical time distribution and shows various interesting behaviors. In the present work, we are interested in how the presence of noise influences the final behavior for the vehicles equipped with ATIS. Similar to [39], we introduce

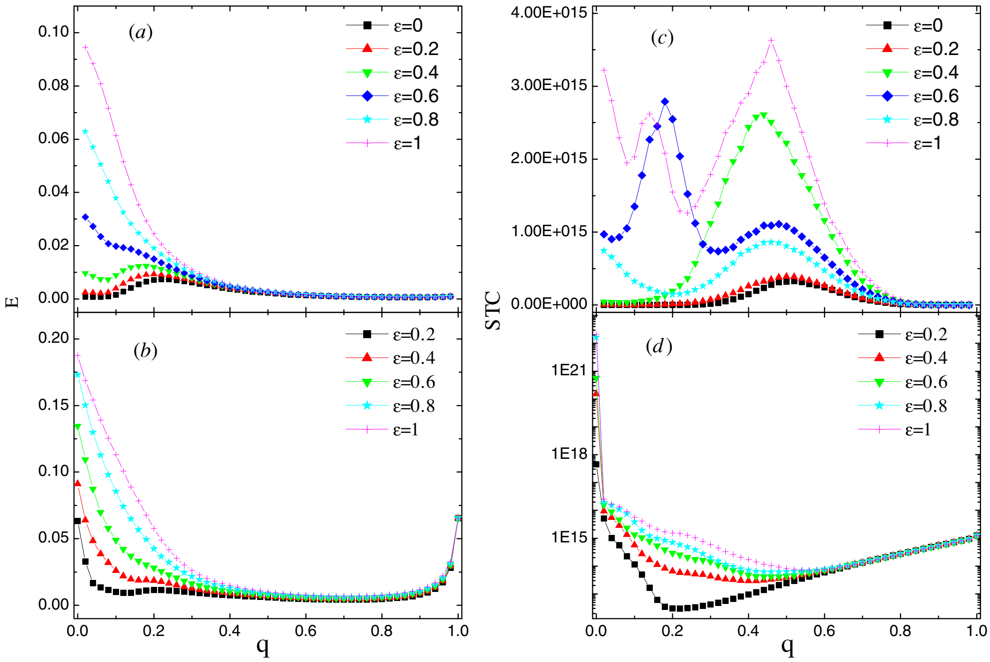

Figure 5. Efficiency E as a function of q in a (*a*) random network and (*b*) scale-free network. System total cost STC as a function of q in a (*c*) random network and (*d*) scale-free network. The network size is $N = 3000$, and the simulations are performed by considering noise effect.

the effect of noise as an erroneous understanding of link cost functions. In detail, at a given error probability ε, the cost of link (i, j) is calculated as w'_{ij}, instead of its correct w_{ij},

$$w'_{ij} = (1 + r)w_{ij}, \tag{4}$$

where r is the uniform random variable with zero mean ($r \in [-1, 1]$). We believe that this erroneous behavior is plausible in reality, since the perfect knowledge for the true value of the cost for each link may not be available. In the limiting case of $\varepsilon = 0$, we recover our error-free results presented above. In figure 5, we report the results for different error probability ε. From figures 5(*a*), (*b*), it is seen that, for small ε, the behavior of the efficiency for both random networks and scale-free networks are qualitatively the same as in figure 1. That is, for random networks, there exists a well-developed efficiency peak and gradual decrease as q is increased; while for scale-free networks, it performs a 'U' form. The peak height of the efficiency is interestingly found to increase as ε is increased, indicating the active effect of the noise for random networks. As ε becomes larger, the peak may vanish. It is clear that at the region of small q the efficiency for both random networks and scale-free networks are effectively improved. From figure 5(*c*), it is obvious that, for random networks, due to the effect of noise, there exist large fluctuations at the region of small q, while such fluctuations vanish for the case of scale-free networks as shown in figure 5(*d*). The reason may be due to the randomization of noise effect. In scale-free networks, most of links and nodes are not very important (We consider 'hub' nodes or links which are to handle or transport more traffic flows to be important). As a result, such important links or nodes are not seriously affected by the effect of noise. Clearly, the noise has a more obvious effect on random networks than scale-free networks, which is similar to the results obtained in [17]. Moreover, from figures 5(*c*), (*d*), one can also find that at the region of small q the system total cost for both random networks and scale-free networks increase as ε is increased, indicating the negative effect of the noise. Obviously, a paradox is also found in terms of noise effect.

3.2. With heterogeneous link cost functions

In this section, $w_{ij}^0 = 1$ and different kinds of C_{ij} are taken into account.

First, following [40], betweenness centrality of the link (i, j), BC_{ij} is used to define its 'practical capacity'. After rules (i) and (ii), the cost of link (i, j), w_{ij} is calculated as

$$w_{ij} = w_{ij}^0 \left(1 + a\left(\frac{x_{ij}}{C_{ij}}\right)^b\right) = w_{ij}^0 \left(1 + a\left(\frac{q \cdot BC_{ij}}{BC_{ij}}\right)^b\right) = w_{ij}^0 (1 + a \cdot q^b). \tag{5}$$

It is clear that each link has identical cost, only depending on parameter q. Then the traffic flow distribution is equivalent to the distribution of betweenness centrality, i.e., corresponding to our model with $q = 1$. The simulation results are not shown here.

Next, we simply assume that the 'practical capacity' of each link depends on the degrees of its endpoints, given as

$$C_{ij} = (k_i \cdot k_j)^\theta, \tag{6}$$

where k_i and k_j is the degree of node i and j, respectively. θ is a control parameter which can adjust the value of link's 'practical capacity'. To make a close correlation to the realistic scenario of link capacity, here we restrain our studies for the values of θ between 0.6 and 1.5.

As shown in figure 6(*a*), it is seen that the behavior of the efficiency for random networks is qualitatively the same as in the inset of figure 1, i.e., the existence of a well-developed efficiency peak. It is natural that the height of the peak quickly increases as θ is increased; moreover, the critical value of parameter q, q_1 is also increased. We conclude that $q_1 = 1$ when θ is large enough. The reason is that larger value of parameter θ means larger link's 'practical capacity'. As a result, the network is always in a free-flow state after rules (i) and (ii) in our model. Then the path routing of vehicles equipped with ATIS may follow that of the vehicles not equipped with ATIS. In other words, each vehicle may travel in terms of free-flow minimum cost, leading to that the optimal value of parameter q corresponding to efficiency peak is equal to 1. From figure 6(*b*), it is obvious that the behavior of the efficiency for scale-free networks is quite different from that shown in figure1. For small θ, there exists a well-developed efficiency peak similar to that for random networks. The heterogeneous link's 'practical capacity' causes that the 'hub' nodes in scale-free networks is less congested after rules (i) and (ii), as compared with the case of homogeneous link cost functions; as a result, the efficiency for scale-free networks is also improved, which is similar to that for random networks.

Similarly, as shown in figure 6(*c*), one can see that the behavior of the system total cost for random networks is qualitatively the same as in the inset of figure 2, and the value of q_2 is seriously affected by θ. From figure 6(*d*), it is clear that, for small θ, the behavior of the system total cost for scale-free networks exhibits a 'bell' shape, similar to that for random networks. The reason may be similar to that mentioned above. Moreover, the system total cost for both random networks and scale-free networks are expected to sharply decreased as θ is increased.

From figure 6, one can also find that, for fixed θ, there exists a paradox between the efficiency and system total cost for both random networks and scale-free networks.

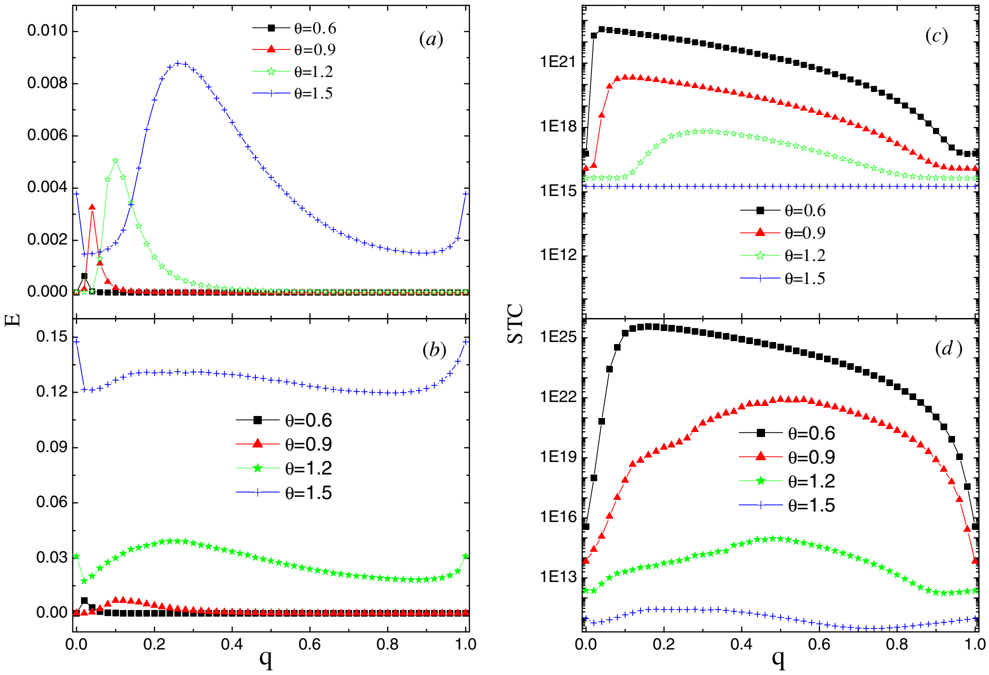

Figure 6. Efficiency E as a function of q in a (a) random network and (b) scale-free network. System total cost STC as a function of q in a (c) random network and (d) scale-free network. The network size is $N = 3000$, and the simulations are performed by considering heterogeneous link cost functions. The 'practical capacity' in link cost functions is given as $C_{ij} = (k_i \cdot k_j)^{\theta}$, where k_i and k_j is the degree of node i and j, respectively, and θ is a control parameter.

3.3. Extended discussions

In our model, when every vehicle is equipped with ATIS, there is no pre-loading of the roads with vehicles without ATIS, and therefore, the vehicles behave identical to the case where no one is equipped with ATIS. And this is very un-realistic. So we extend our model by inserting the vehicles into the network at a constant rate R in time, where $1 - q$ fraction of the vehicles are equipped with ATIS and the rest are not. The path routing strategy of each vehicle follows the rules in our model. That is, for vehicles with ATIS entering the system at time t, the route to the destination is chosen according to the flow on the links at t, whereas for vehicles without ATIS simply according to the network topology. In the following, we focus on investigating the effects of noise and cost functions, so we fix $w_{ij}^0 = 1$, $N = 1000$ and $R = 0.05$ (which is normalized to unity), other parameters are similar to the above sections.

First, we study the case of homogeneous link cost functions. From figure 7, one can find that the efficiency and system total cost for both random networks and scale-free networks increase as q is increased, which is different from the above results. However, it indicates that a paradox between the efficiency and system total cost is recovered. Similarly, as shown in figure 8, for random networks, it indicates an active effect of the noise for efficiency, while the noise performs a negative effect for system total cost, i.e., the system total cost obviously increases as ε is increased at the region of small q; for scale-free networks, the efficiency slightly increases as ε is increased, while the system total cost increases as ε is increased at the region of small q. It is recovered that the noise has a more obvious effect on random networks

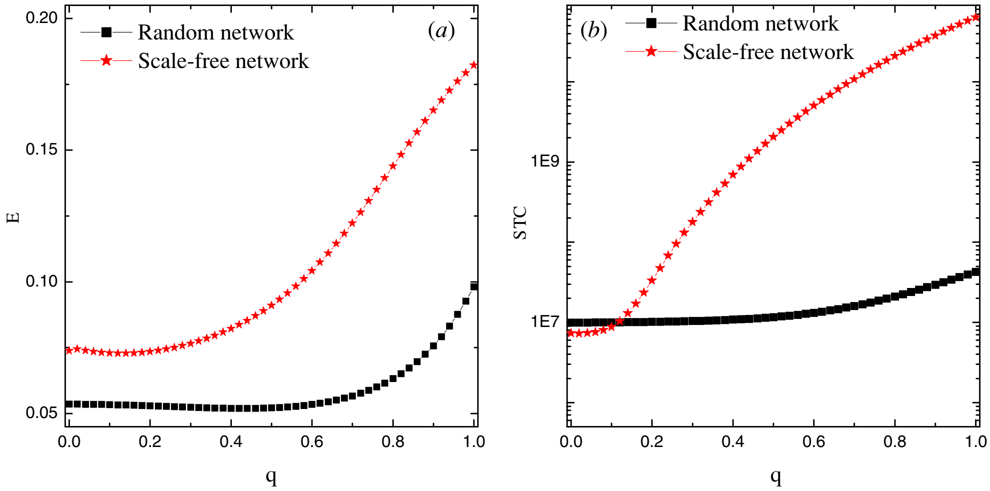

Figure 7. (a) Efficiency E and (b) system total cost STC as a function of q for the extended traffic model.

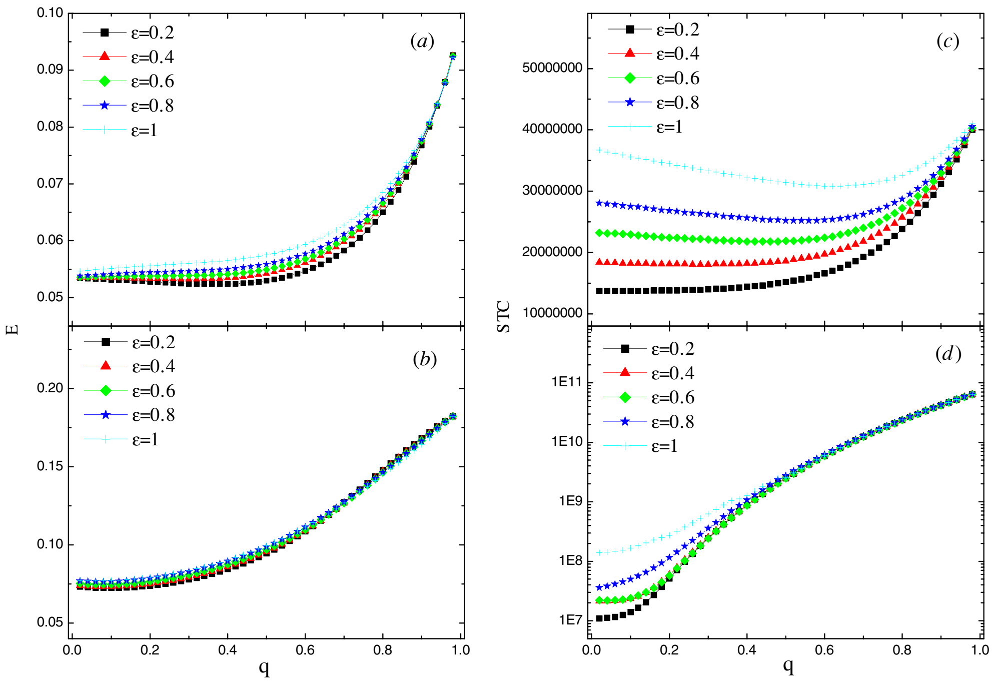

Figure 8. As shown in figure 5, but for the extended traffic model.

than scale-free networks. Moreover, a paradox between the efficiency and system total cost is recovered in terms of noise effect.

Next, we investigate the case of heterogeneous link cost functions. Figure 9 shows that, for both random networks and scale-free networks, both the efficiency and system total cost

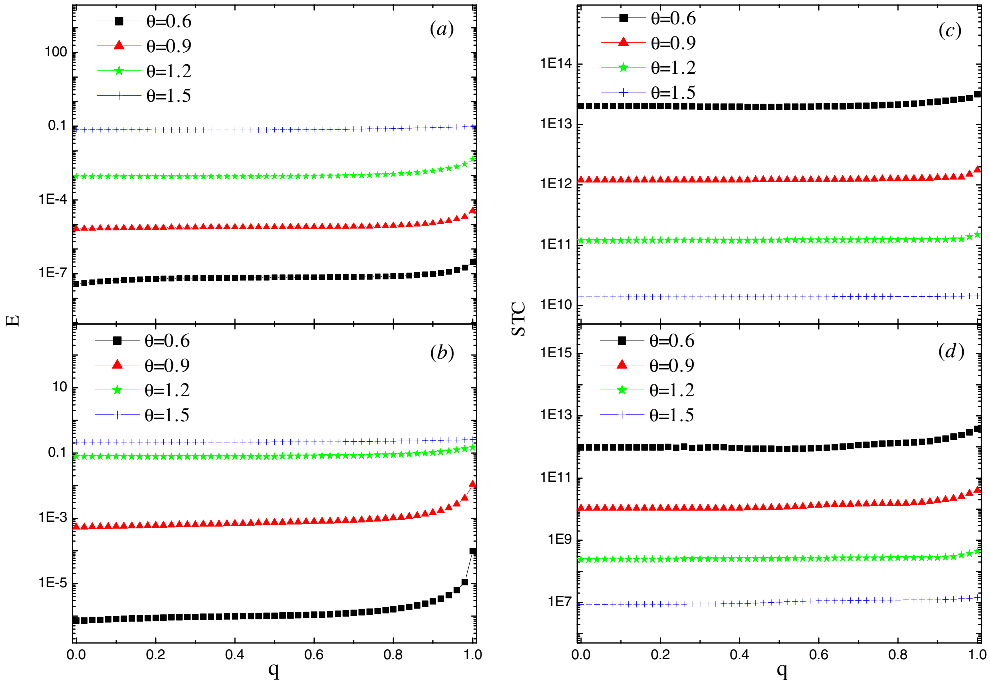

Figure 9. As shown in figure 6, but for the extended traffic model.

are expected to be seriously affected by θ, which is defined in the above section. Moreover, for fixed θ, a paradox between the efficiency and system total cost is recovered.

4. Summaries and conclusions

In this paper, we propose and study some properties of a traffic dynamics model in complex networks by considering ATIS. The results show that network topology plays an important role in determining traffic dynamics. According to our model, the burden of 'hub' nodes in the scale-free network is largely relieved. As a result, the system total cost is decreased. However, the random network exhibits opposite behavior.

Moreover, if the ATIS is properly equipped, the efficiency for the random network is effectively improved, while the sale-free network undergoes inefficient behavior. It indicates that there exists a paradox between the efficiency and system total cost. Furthermore, we perform simulations by investigating different kinds of link cost functions, and the paradox is recovered. Finally, we extend our model, and also find the existence of the paradox. We hope our results may be helpful for the network designers or network managers to better understand traffic dynamics in complex networks.

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